ABSTRACT

This thesis delves into the exploration of transcendence theory, focusing on the unique properties of specific numbers and the behavior of mathematical functions such as modular forms. The research is centered on applying these transcendence concepts to solve Diophantine equations, particularly those associated with sequences following a structured pattern, such as Lucas sequences. The primary objective is to address the problem of solving complex Diophantine equations within Lucas sequences, leveraging transcendence theory and computational methods. The materials and methods employed include rigorous mathematical analysis and Baker's theory on nonzero linear forms in logarithms.

First, I establish the non–existence of positive integer solutions to an exponential Diophantine equation within Lucas sequences, resolving an open question in this area. Second, I provide insights into prime factorization within $k-$generalized Lucas numbers, identifying lower bounds for the largest prime factors and highlighting instances where these numbers can be expressed as a product of small primes. Additionally, the analysis reveals specific pairs within $k-$generalized Lucas numbers that exhibit multiplicatively dependent behavior, shedding light on their arithmetic properties.

Furthermore, the absence of Lucas numbers that are palindromic concatenations of two distinct repdigits is demonstrated, leaving open avenues for further exploration in $k-$generalized Lucas numbers. These results not only contribute to the understanding of Lucas sequences and their applications in solving complex Diophantine equations but also pave the way for future research in this area.

Supervisors

Prof. Florian Luca	Dr. Mahadi Ddamulira	Prof. Juma Kasozi
BSc.Math.(UAIC),	BSc.Ed.(Mak), MSc.(KsTU),	BSc.Ed.(Mak),
PhD(UAF).	Dip.Math.(UniTS),	MSc.(Mak), MSc.(Uz),
Mathematics Division	PhD(Graz).	PhD(Mak–UiB).
Stellenbosch University	Department of Mathematics	Department of Mathematics
South Africa	Makerere University	Makerere University